

Mathematics

Advanced Subsidiary GCE

Unit **4722**: Core Mathematics 2

Mark Scheme for June 2012

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of candidates of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, OCR Nationals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support, which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

© OCR 2012

Any enquiries about publications should be addressed to:

OCR Publications
PO Box 5050
Annesley
NOTTINGHAM
NG15 0DL

Telephone: 0870 770 6622
Facsimile: 01223 552610
E-mail: publications@ocr.org.uk

1. Annotations

Annotation in scoris	Meaning
✓ and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	

Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

2. Subject-specific Marking Instructions

- a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

- b An element of professional judgement is required in the marking of any written paper. Remember that the mark is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks. Marks must not be judged on the answer alone, and answers that are given in the question, especially, must be validly supported. Steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the method awarded marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several candidates are involved) you should contact your Team Leader.

- c The following types of marks are available.

M

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the method allowed for the award of an M mark may be specified.

A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be awarded unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B

Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would be the case where a candidate passes through the correct answer as part of a wrong argument.

- d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may be that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no marks should sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the marks are implied and full credit must be given.
- e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previous results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, the acceptable alternatives will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks are given for 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question if it is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

- f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures being the norm. Small variations in the degree of accuracy to which an answer is given (eg 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should not result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a matter for discussion should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, the examiner should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what the candidate has written in the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

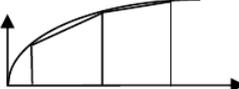
h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question are unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied which is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question	Answer	Marks	Guidance
2 (i)	$\int (x^2 - 2x + 5) dx = \frac{1}{3}x^3 - x^2 + 5x + c$	M1 A1 A1 [3]	Attempt integration Obtain two correct (algebraic) terms Obtain fully correct expression (allow no + c) An increase in power by 1 Allow if the +5 disappears. Allow if the coefficient of x^2 isn't y Allow if the coefficient of x^2 isn't y A0 if integral sign or dx still present allow $\int = \dots$. A0 if a list of terms rather than an
2 (ii)	$y = \frac{1}{3}x^3 - x^2 + 5x + c$ $11 = 9 - 9 + 15 + c \Rightarrow c = -4$ hence $y = \frac{1}{3}x^3 - x^2 + 5x - 4$	M1* M1d* A1 [3]	State or imply $y =$ their integral from (i) Attempt to find c using (3, 11) Obtain $y = \frac{1}{3}x^3 - x^2 + 5x - 4$ Must have come from integration of \dots have been gained in part (i). Allow slips when transferring expression Can still get this M1 if no + c. The y does not have to be explicit eg $11 = F(3)$ (but not by $3 = F(11)$) Using definite integration with limits M0 if they start with $y =$ their integral attempt to use $y - 11 = m(x - 3)$. This gains no credit. Need to get as far as attempting c . M1 could be implied by eg $11 = 9 - 9 + 15 + c$ attempt to include a constant to balance though + c never actually seen. M0 if no + c seen or implied. M0 if using $x = 11, y = 3$. Coeff of x^2 now needs to be simplified Must be an equation ie $y = \dots$, so A 'equation = ...' Allow aef, such as $3y = x^3 - 3x^2 + \dots$

Question			Answer	Marks	Guidance	
5	(a)	(i)	$u_2 = \frac{1}{2}$	B1	State $\frac{1}{2}$	Allow 0.5 or $\frac{2}{4}$.
			$u_3 = 4$	B1 FT	State 4, following their u_2	Follow through on their u_2 (B0 for $\frac{2}{0.5}$, $\frac{2}{\frac{1}{2}}$ etc.
5	(a)	(ii)	periodic / alternating / repeating / oscillating / cyclic	B1	Any correct description	Allow associated words eg 'repetitive'. Must be a mathematical term rather than 'it changes between 4 and $\frac{1}{2}$ ' or 'terms are $\frac{1}{2}$ '. Mark independently of any values. Ignore irrelevant terms (eg 'recursive') and additional incorrect terms (eg 'geometric').
5	(b)		$a + 8d = 18$	B1	State $a + 8d = 18$	Allow any equivalent, including $8d + a = 18$. Must be correct when seen – can't be stated but with incorrect a substituted.
			$\frac{9}{2}(2a + 8d) = 72$	B1	State $\frac{9}{2}(2a + 8d) = 72$	Allow any equivalent, including $9(2a + 8d) = 72$. Must be correct when seen – as above.
			$a + 8d = 18$ and $2a + 8d = 16$	M1	Attempt to solve simultaneously	M1 is awarded for eliminating a variable from two equations in a and d , from attempting to solve for a or d at $S_9 = 72$ (formulas must be recognised but not necessarily correct). Don't award for balancing equations, then there is no need to subtract (but allow $a = 2$). If substituting then allow sign errors. Do not award for non-operational errors (eg $a = \frac{18}{8d}$).
			$a = -2, d = \frac{5}{2}$	A1	Obtain either $a = -2$ or $d = \frac{5}{2}$	A1 is given for the first correct value. Allow $d = 2\frac{1}{2}$ or 2.5, but not unsimplified.
				A1	Obtain both $a = -2, d = \frac{5}{2}$	A1 is given for obtaining second correct value. Allow $d = 2\frac{1}{2}$ or 2.5, but not unsimplified.
				[5]		

Question	Answer	Marks	Guidance	
	<p>OR</p> $\frac{9}{2}(a + 18) = 72$ $a = -2$ $-2 + 8d = 18 \text{ or}$ $\frac{9}{2}(-4 + 8d) = 72$ $d = \frac{5}{2}$	<p>B1*</p> <p>B1d*</p> <p>M1</p> <p>A1FT</p> <p>A1</p>	<p>Alternative method using $\frac{n}{2}(a + 1)$</p> <p>State $\frac{9}{2}(a + 18) = 72$</p> <p>Obtain $a = -2$</p> <p>Attempt use of either u_9 or S_9</p> <p>Obtain correct equation, following their a</p> <p>Obtain $d = \frac{5}{2}$</p>	<p>NB If using $a + (n - 1)d$ and solving simultaneous equations per scheme below.</p> <p>Allow any equivalent. Award B1 as soon as seen correct, error.</p> <p>Must come from correct equation.</p> <p>Must be attempting either $u_9 = 18$ Must be using correct formula.</p> <p>Allow any equivalent, including u_9</p> <p>Allow $d = 2\frac{1}{2}$ or 2.5, but not unsimplified</p>

Question		Answer	Marks	Guidance	
6	(i)	$0.5 \times 4 \times (4\sqrt{1} + 8\sqrt{5} + 4\sqrt{9})$ $= 2(16 + 8\sqrt{5})$ $= 32 + 16\sqrt{5} \quad \mathbf{AG}$	M1* M1d* A1 [3]	Attempt y -values at $x = 1, 5, 9$ only Attempt correct trapezium rule, inc $h = 4$ Obtain $32 + 16\sqrt{5}$	Must be using y , not x at Allow slips eg $\sqrt{(4x)}$ as 1 Allow decimal equiv for y_1 Allow M1 for 4, 20, 72 (ie om. M0 if other y -values found (unles. Correct structure, including 'big b Allow 2 used for $\frac{1}{2} h$ – no need fo Allow slips when calculating y val must be correct. Could use two separate trapezia. Must come from exact working, so found in decimals (67.777...) whic same as $32 + 16\sqrt{5}$. However, isw first, and then decimal equiv stated
6	(ii)	 <p>Curve is above tops of trapezia</p>	B1* B1d* [2]	Sketch showing correct graph of $y = 4\sqrt{x}$ and two trapezia (allow if only tops of trapezia seen as chords) Reason comparing the tops of trapezia to the curve, or referring to the gap between the trapezia and the curve	Correct graph shown, existing for Exactly two trapezia must be show widths, with top vertices on the cu Must refer to the tops of the trapez below curve' (ie 'top' not used). Allow 'trapezium' rather than 'trap Could shade gaps on their diagram B0 for 'some area not calculated' r Concave / convex is B0, as is com B1 for decreasing gradient (but B0 B0 (rather than isw) if explanation No sketch is B0, irrespective of ex SR B1 for correct explanation, and graph of $y = 4\sqrt{x}$ for $1 \leq x \leq 9$ but t (eg curvature / y -intercept / not jus

Question	Answer	Marks	Guidance	
6 (iii)	$\int_1^9 4x^{\frac{1}{2}} dx = \left[\frac{8}{3} x^{\frac{3}{2}} \right]_1^9$ $= 72 - \frac{8}{3}$ $= 69\frac{1}{3}$	M1	Obtain $k x^{\frac{3}{2}}$	Any numerical k , incl Any exact equiv for the
		A1	Obtain $\frac{8}{3} x^{\frac{3}{2}}$	Allow unsimplified coefficient, Allow non exact decimal ie 2.7, 2. Allow $+c$.
		M1	Attempt correct use of limits	Must be $F(9) - F(1)$ ie subtraction order. Allow use in any function other th from differentiation.
		A1	Obtain $69\frac{1}{3}$, or any exact equiv	Allow processing errors eg $(\frac{8}{3} \times 9)^{\frac{1}{3}}$ Allow improper fraction, or recurr A0 for 69.333.... A0 for $69\frac{1}{3} + c$.
		[4]		Answer only is 0/4.

Question			Answer	Marks	Guidance	
7	(a)	(i)	$\cos \alpha = \frac{5}{\sqrt{29}}$	M1	Attempt $\cos \alpha$	Could draw triangle and use Pythagoras to find hypotenuse, or use trig identities. Must get as far as attempting $\cos \alpha$. Must be working in exact values for trig ratios. Must be using correct ratios for triangle.
		A1		Obtain $\frac{5}{\sqrt{29}}$	Allow any exact equiv, including $\frac{5\sqrt{29}}{29}$ or $\sqrt{\frac{25}{29}}$ isw if decimal equiv subsequently Answer only gets full credit.	
		[2]			SR B1 for exact answer following	
7	(a)	(ii)	$\cos \beta = \frac{-\sqrt{40}}{7}$	M1	Attempt $\cos \beta$	Could draw triangle and use Pythagoras to find adjacent, or use trig identities. Must get as far as attempting $\cos \beta$. Must be working in exact values for trig ratios. Must be using correct ratios for triangle.
		A1		Obtain $\frac{\sqrt{40}}{7}$	Allow any exact equiv, including $\frac{\sqrt{10}}{7}$ Allow $\pm \frac{\sqrt{40}}{7}$ (from using $\cos^2 x = \dots$) isw if decimal equiv subsequently Answer only gets M1A1.	
		A1 FT		Obtain $\frac{-\sqrt{40}}{7}$, or -ve of their exact numerical value for $\cos \beta$	A1 FT can only be awarded following M1A1 isw if decimal equiv subsequently Answer only gets full credit.	
				[3]		SR B1 for $\frac{\sqrt{40}}{7}$, or equiv, following M1A1 SR B2 for $\frac{-\sqrt{40}}{7}$, or equiv, following M1A1 SR B1 for decimal answer in range

Question		Answer	Marks	Guidance	
7	(b)	$\frac{\sin \gamma}{6} = \frac{\sin 60}{8}$	M1*	Attempt use of correct sine rule	Must be correct sine rule substitute values in – no
			M1d*	Use $\sin 60^\circ = \frac{\sqrt{3}}{2}$	Could be implied eg $\frac{6}{\sin \gamma} = \frac{16}{3}$
		$\sin \gamma = \frac{3\sqrt{3}}{8}$	A1	Obtain $\sin \gamma$ as $\frac{3\sqrt{3}}{8}$	Must be seen simplified to this, or isw if decimal equiv subsequently isw any attempt to find the angle.
			[3]		A0 if only ever seen as $\sin^{-1} \frac{3\sqrt{3}}{8}$

Question	Answer	Marks	Guidance
8 (i)	$f(2) = 8 + 2a - 6 + 2b = 0$ $g(2) = 24 + 4 + 10a + 4b = 0$	M1	Attempt at least one of $f(2)$, $g(2)$
		M1	Equate at least one of $f(2)$ and $g(2)$ to 0
	$2a + 2b = -2, 5a + 2b = -14$	A1	Obtain two correct equations in a and b
	hence $3a = -12$	M1	Attempt to find a (or b) from two simultaneous eqns
	so $a = -4$ AG	A1	Obtain $a = -4$, with necessary working shown
	$b = 3$	A1	Obtain $b = 3$
		[6]	

Allow for substituting into one equation to simplify at this stage.
 Division – complete attempt
 Coeff matching - attempt all 3

Just need to equate their substituted values (writing eg $f(2) = 0$ is not enough).
 It could be implied by later working to solve equations.
 Division - equating their remainders
 Coeff matching – equate constant terms

Could be unsimplified equations.
 Could be $8a + 2b = -26$ (from $f(2)$)

Equations must come from attempting to solve $g(2) = 0, f(2) = g(2)$.
 M1 is awarded for eliminating a or b to solve for the other.
 allow sign slips only.
 Most will attempt a first, but they will find b from their simultaneous equations

If finding b first, then must show working to find a (unless earlier shown, eg $a = -1 - b$).

Correct working only

SR Assuming $a = -4$
Either use this scheme, or the one for finding a first, or elements from both
 M1 Attempt either $f(2)$ or $g(2)$
 M1 Equate $f(2)$ or $g(2)$ to 0 (also allow $f(2) = g(2)$)
 A1 Obtain $b = 3$
 A1 Use second equation to confirm $a = -4$

Question			Answer	Marks	Guidance	
9	(a)	(i)	$u_4 = \log_2 27 + 3\log_2 x$ $= \log_2 27 + \log_2 x^3$ $= \log_2(27x^3)$ AG	M1 M1 A1 [3]	Use $u_4 = a + 3d$ Use $b \log a = \log a^b$ on $3\log_2 x$ Show $\log_2(27x^3)$ convincingly	Allow missing / incorrect / inconsistent Starting with $\log_2 27 + \log_2 x^3$ Starting with $\log_2 27 \times 3\log_2 x$ (below). Starting with $\log_2 27 + \log_2 x + \log_2 x^2$ credit. u_4 must still be shown as two terms Could get M1 if using $a + 4d$. Could get M1 for $\log_2 27 \times 3\log_2 x$ $\log_2 27 \times 3\log_2 x = \log_2 27 + \log_2 x^3$ Allow missing / incorrect / inconsistent Can go straight from $\log_2 27 + \log_2 x^3$ CWO, including using base 2 through SR – finding consecutive terms (explicit) B1 for $u_2 = \log_2 27 + \log_2 x = \log_2 27x$ B1 for $u_3 = \log_2 27x + \log_2 x = \log_2 27x^2$ B1 for $u_4 = \log_2 27x^2 + \log_2 x = \log_2 27x^3$
		(ii)	$27x^3 = 2^6$ $x = \sqrt[4]{3}$	B1* B1d* [2]	State correct equation no longer involving $\log_2 x$ Obtain $\sqrt[4]{3}$	Equation could still involve constant or $\log_2 3$. Allow truncated or rounded decimal Must be $\sqrt[4]{3}$, $1\frac{1}{3}$ or an exact recurring (1.333...). A0 if cube root still present. Working must be exact, so sight of $\sqrt[4]{3}$ used is B0, even if final answer is $\sqrt[4]{3}$. Answer only gets full credit.

Question			Answer	Marks	Guidance	
9	(b)	(i)	$\frac{1}{2} < y < 2$	M1	Identify at least one of $\frac{1}{2}$ and 2 as end-points	Only one end-point required. Ignore if additional inequalities used. Ignore any signs used.
				A1	Obtain $\frac{1}{2} < y < 2$	Not two separate inequalities, unless stated. A0 for $\frac{1}{2} \leq y \leq 2$.
9	(b)	(ii)	$\frac{\log_2 27}{1 - \log_2 y} = 3$ $\log_2 27 = 3 - 3\log_2 y$ $\log_2 27 = 3 - \log_2 y^3$ $\log_2(27y^3) = 3$ $27y^3 = 8$ $y^3 = \frac{8}{27}$ $y = \frac{2}{3}$	B1	State $\frac{\log_2 27}{1 - \log_2 y} = 3$	Allow B1 if no base stated, but B0 if not. Must be equated to 3 for B1.
				M1*	Attempt to rearrange equation to $\log_2 f(y) = k$	Must be using $\frac{\log_2 27}{\pm 1 \pm \log_2 y}$ (but allow any sign). Allow at most 2 manipulation errors, including muddles, or slips when expanding brackets. Other errors (eg incorrect use of log laws) are A0.
				M1d*	Use $f(y) = 2^k$ as inverse of $\log_2 f(y) = k$	Must have first been arranged to $f(y) = 2^k$. No need to go any further than stating $f(y) = 2^k$.
				A1*	Obtain correct exact equation no longer involving $\log_2 y$	Equation could still involve constants, but not $\log_2 3$. Sight of decimals used is A0, even if correct.
				A1d*	Obtain $\frac{2}{3}$	Allow equiv recurring decimal, but not $\frac{2}{3}$. A0 if still cube root present.
			[5]		SR answer only is B3 Correct $S_\infty = 3$, then answer with n .	

Guidance for marking C2**Accuracy**

Allow answers to 3sf or better, unless an integer is specified or clearly required.

Answers to 2 sf are penalised, unless stated otherwise in the mark scheme.

3sf is sometimes explicitly specified in a question - this is telling candidates that a decimal is required rather than an exact answer eg in should not be penalised unless stated in mark scheme.

If more than 3sf is given, allow the marks for an answer that falls within the guidance given in the mark scheme, with no obvious errors.

Extra solutions

Candidates will usually be penalised if an extra, incorrect, solution is given. However, in trigonometry questions only look at solutions ignore any others, correct or incorrect.

Solving equations

With simultaneous equations, the method mark is given for eliminating one variable. Any valid method is allowed ie balancing or substitution, substitution only if at least one is non-linear.

Solving quadratic equations

Factorising - candidates must get as far as factorising into two brackets which, on expansion, would give the correct coefficient of x^2 and two coefficients. This method is only credited if it is possible to factorise the quadratic – if the roots are surds then candidates are expected to use the quadratic formula or complete the square.

Completing the square - candidates must get as far as $(x + p) = \pm \sqrt{q}$, with reasonable attempts at p and q .

Using the formula - candidates need to substitute values into the formula, with some attempt at evaluation (eg calculating $4ac$). Sign slip in $4ac$, but all other aspects of the formula must be seen correct, either algebraic or numerical. The division line must extend under the c implied by later working). If the algebraic formula is quoted then candidates are allowed to make one slip when substituting their values. $2a$ as long as it has been seen earlier.

OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
Cambridge
CB1 2EU

OCR Customer Contact Centre

Education and Learning

Telephone: 01223 553998

Facsimile: 01223 552627

Email: general.qualifications@ocr.org.uk

www.ocr.org.uk

For staff training purposes and as part of our quality assurance programme your call may be recorded or monitored

Oxford Cambridge and RSA Examinations
is a Company Limited by Guarantee
Registered in England
Registered Office; 1 Hills Road, Cambridge, CB1 2EU
Registered Company Number: 3484466
OCR is an exempt Charity

OCR (Oxford Cambridge and RSA Examinations)
Head office
Telephone: 01223 552552
Facsimile: 01223 552553

© OCR 2012

